

REQUIREMENTS FOR A MARINE GEOID COMPATIBLE WITH  
GEOID DEDUCIBLE FROM SATELLITE ALTIMETRY

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Because bewildering confusion about the geoid exists, the first part of the paper is a systematic review of the concept of the geoid and the various geodetic techniques and associated data employed in the physical determination of the geoid. The deficiencies in theory, data, and practical computational procedures that have made the physical determination of the geoid with true scale, shape, and absolute orientation an elusive target are outlined. The potential of satellite altimetry, in combination with adequate ground support and "sea-truth", to resolve the accurate determination of a global marine geoid (the geoid in the oceans) and other peripheral benefits associated with ocean physics is briefly restated. Attention is drawn to the controversy as to the validity of using a best fitting ellipsoid ( $f = 1/298.25$ ) instead of an equilibrium ellipsoid ( $f = 1/299.67$ ) in all gravimetric work for computing gravity anomalies and the geoid, and for geophysical interpretations from gravity surveys.

Marine gravity measurements alone cannot adequately furnish the required geodetic sea-truth. The paper indicates the "how and "why" a combination of marine astrogravimetry and marine geodetic acoustic techniques is the best approach to meet the requirements for "sea-truth" (segments of the absolute marine geoid in test areas) compatible with the geoid deducible from satellite altimetry. Table 4 at the end of the paper contains a summary of the findings.

## 1. INTRODUCTION

The geoid is that equipotential surface in the gravity field of the earth which most nearly coincides with the undisturbed mean sea level. In spite of this exactness of definition, the physical determination of the true geoid remains an elusive target to geodesists. Consequently, many concepts and classes of concepts concerning how it should be physically determined have arisen. In scale, shape, and orientation, each class of geoids has little in common with another class. Even within the class, the various geoids differ and depend on many factors such as (1) the parameters of the reference ellipsoid which, for convenience, geodesists always associate with each geoid, (2) the measuring technique, the measurements and their reductions in theory and in practice, (3) the quantity and quality of data, and (4) the datum origin of the geodetic system.

Because the geoid is an irregular surface which does not exactly conform to any known geometric figure, it is geometrically defined by its physical departures from a chosen regular figure which is usually a reference ellipsoid. In some methods, the departures are determined by linear and angular measurements while in others these departures are synthesized from gravity anomalies integrated all over the earth's surface or a combination of both. The latest generation of geoids is deduced from the analysis of the dynamics of satellite orbits or a combination of gravimetry and satellite orbit analysis.

To amplify the dissimilarity between the various geoids, the concepts and data for their determinations and the physical meaning and nature of what is determined will now be reviewed. The objective is to demonstrate why anything that currently goes by the name marine geoid should neither be expected to be compatible in scale,

shape, and orientation with the geoid determinable from satellite altimetry nor be used as a means of geodetic absolute verification or calibration of satellite altimetry. Besides, the immediate direct results of the altimeter data are average sea surface topography and not the required geoid.

Having determined that the best approximation and convenient geometric figure for the geoid is a rotational ellipsoid, geodesists have continued to expend a lot of energy to determine the size and shape of the reference ellipsoid most desirable for geodetic computations. Numerous determinations of reference ellipsoids exist [Mueller, et al, 1966] but will not be discussed to spare the reader further complications. However, one important complication usually ignored but which was emphasized again at the 1967 International Symposium on the Figure of the Earth and Refraction in Vienna is that the best fitting ellipsoids, flattening of about  $1/298.25$ , in geodetic use significantly differ from the hydrostatic or equilibrium ellipsoid, flattening of about  $1/299.67$ . O'Keefe [1967] strongly suggests that all gravimetric work for computing anomalies and the geoid, and for geophysical interpretations from gravity surveys should refer not to the best fitting ellipsoid but to the hydrostatic or equilibrium ellipsoid. Fischer [1967] and Gaposchkin and Lambeck [1970] have the first practical computations for examining this unresolved complication.

Discussions about the quasigeoid [Molodenskii, et al, 1962] as a substitute to bypass certain difficulties concerning the geoid is avoided here because in the oceans, the geoid and quasigeoid coincide [Heiskanen and Moritz, 1967].

Figure 1 is a vertical section depicting a typical relationship between the geoid and an ellipsoid. The general nonparallelity between the geoid and the ellipsoid implies that in the same location, the normals to the two surfaces intersect at an angle,  $\epsilon$ , called the deflection of the vertical in that plane. The geoidal undulation,  $N$ , is the linear vertical separation between the geoid and the ellipsoid.

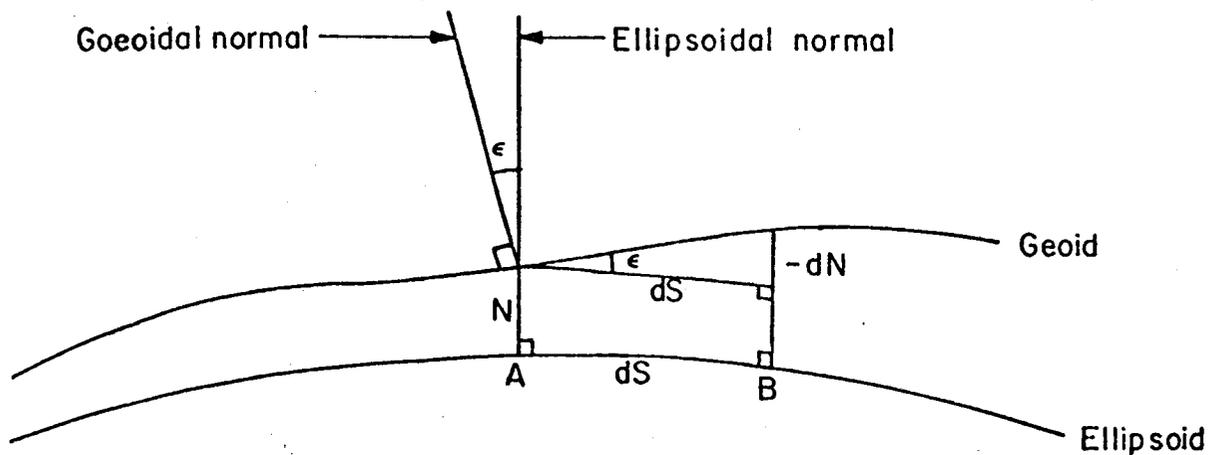


FIGURE 1. SCHEMATIC RELATION BETWEEN GEOID AND ELLIPSOID

With reference to Figure 1, the increment  $dN$  in  $N$ , over the distance  $dS$  is given, according to Helmert as stated in [Heiskanen and Moritz, 1967] by

$$dN = - \epsilon dS \quad (1)$$

which, on integrating, results in

$$N_B = N_A - \int_A^B \epsilon dS \quad , \quad (2)$$

where  $\epsilon$  is the deflection of the vertical in any arbitrary azimuth,  $\alpha$ , measured clockwise from the north, and given by

$$\epsilon = \xi \cos \alpha + \eta \sin \alpha \quad , \quad (3)$$

where  $\xi$  and  $\eta$  are the deflection components in the meridian and prime vertical respectively. If the various values of  $\epsilon$  for different places in an area are determined, then by the use of Equation (2) the geoid of the area can be computed.

Some of the most important categories of the geoid and their characteristics are described below. For each class of geoids, the theory

implied and type of data employed, and the deficiencies in the theory, the quality and quantity of data currently in use, will be outlined. The expectation to map sea surface topography and eventually the marine geoid is widely known. The need for test areas with reliable "ground or sea-truth" including geoidal profiles with accurate scale, shape, and orientation is also widely recognized but the methodology for meeting this need such as by gravity data alone is indicated to be grossly inadequate.

The geodetic processing of reliable satellite altimetry data should determine the true geoid with absolute orientation, correct scale, and detailed features of the true shape. The paper advocates the use of a combination of astrogravimetry [Molodenskii, et al, 1962] and marine geodetic-acoustic techniques [Mourad, et al, 1970b] as the most expedient means for establishing marine geoidal profiles compatible with those deducible from satellite altimetry at sea. Marine geoid is used to denote the geoid in the oceans as distinct from continental geoid computed on land. For the most meaningful and reliable geodetic deductions from satellite altimetry, two calibrations must be distinguished. The first is a hardware calibration to ensure that an altimeter range indicated as xx meters is indeed xx meters to within the instrument's assigned accuracy. The second is a geodetic calibration or control required if a geoid with true scale, true shape, and absolute orientation is to be deduced from satellite altimetry. This paper is addressed to the requirements of the geodetic calibration. This is highly relevant because the altimeter readings are not made to the surface of the actual geoid but to some unknown "electromagnetic mean surface" as discussed in Section 3.

## 2. CLASSIFICATION OF GEOIDS

Several methods have been developed and/or used in determining the geoid. Examples of these methods which are described here include: (1) astrogeodetic, (2) inertial, (3) gravimetric, (4) satellites, (5) altimetry, and (6) astrogravimetric methods.

## 2.1 Astrogeodetic Geoids

### 2.11 The Classical Astrogeodetic Geoid

The coordinates of any point on the surface of the earth can be depicted by its geodetic latitude,  $\varphi$ ; geodetic longitude,  $\lambda$ , and geodetic height,  $h$ , as determined by classical terrestrial geodesy, where  $h$  is the height of the point above the reference ellipsoid. The same point, referenced to the geoid, can be depicted by the astronomic latitude,  $\Phi$ , astronomic longitude,  $\Lambda$ , and orthometric height,  $H$ , above the geoid. The interrelationship between these parameters is generally expressed by

$$\xi = \Phi - \varphi \quad (4)$$

$$\eta = (\Lambda - \lambda) \cos\varphi \quad (5)$$

$$N = h - H \quad (6)$$

The orthometric height is approximately the geometric height above mean sea level, measured along the geoidal normal [Heiskanen and Moritz, 1967].

The geoid determined by inserting the differences between the astronomic and geodetic coordinates of the same point through the use of Equations (4) and (5) into Equations (3) and (2), is termed astrogeodetic.

The astronomical latitude and longitude are determined directly by observing stars. Within the limits of observational accuracy, the accuracy of star coordinates in space, and the adequate application of all corrections involved in astronomical measurements and reductions, the astronomical latitude and longitude of a place are unique. In sharp contrast, the geodetic coordinates of any point could be made unique but currently most are not "unique" but depend on the geodetic datum. The size and orientation of each datum reference ellipsoid is different and the position of the reference ellipsoid with respect to any unique point such as the center of mass of the earth remained unknown until the advent of dynamic satellite triangulation which has not yet resolved the problem satisfactorily. This will be discussed later using computations from Veis [1965, 1968] and Lambeck [1971]. Theoretical studies by Rapp [1970c] and Fubara [1971] and the work of Mather [1970, 1971] offer resolutions to this problem.

Consequently, the ensuing components of the deflection are not unique

- (1) To each datum, there is a different astrogeodetic geoid.
- (2) In shape, size, and orientation, astrogeodetic geoids on different datums are incompatible.
- (3) Because of several weaknesses in current astrogeodetic practice, falsely exaggerated geoidal undulations and hence false geoidal tilts are progressively perpetrated the further a place is from the datum origin.

As shown by Fischer [1959], at long distances from the datum origin, computed geoidal undulation of 200 to 300 meters exist. Even after the application of the theoretically necessary Molodenskii's correction [Molodenskii, et al, 1962], which amounted to -60 m at a place 80° south of the North American Datum (NAD 1927), the geoidal height was 260 m [Fischer, 1959].

These inherent qualities of the classical astrogeodetic geoid and its rapid deterioration in shape precludes its use as a means of absolute verification of any other type of geoid without translations and transformations which are described later. The parameters for these reconciliations are still not accurately known. Above all, computation of astrogeodetic geoids has usually been limited to the continents because of the difficulties in determining usable geodetic and astronomic coordinates at sea. In this respect, von Arx [1966] made a valuable pioneering effort but also added a caution which is usually not remembered that as he put it: "The accuracy attainable is barely comparable with that achieved by Eratosthenes 2 millennia ago when he estimated the circumference of the earth".

## 2.12 Astrosatellite Geoid

There are many methods of determining  $\epsilon$  or  $\xi$  and  $\eta$  which, in turn, are used to compute a geoid, using Equation (2). When the geodetic coordinates  $\phi$ ,  $\lambda$ , and  $h$  used are obtained from satellite fixes instead of terrestrial triangulation, traverse, etc., the resultant geoid can be termed astro-satellite. Satellite derived coordinates are supposedly known in a

geocentric system to an accuracy between  $\pm 5$  to  $\pm 20$  meters. Based on absolute geocentric coordinates, an astrosatellite geoid or any other geoid computed by Equation (2) is in absolute position if and only if at the starting point of the integration the absolute geoidal undulation is known.

In principle, the shape and size of such an astrosatellite geoid and the geoid deducible from satellite altimetry should be identical. In practice on land, the precision of each of the geodetic coordinates from satellite fixes is at best about  $\pm 5$  meters. At sea, a geodetic position fix, as determined from improved Doppler satellite receivers, could be obtained to perhaps  $\pm 10$  to  $\pm 20$  m if one used a fixed station defined by a ship positioned over ocean-bottom transponders where many satellite passes are taken and reduced to the same point. Furthermore, long arcs of astrosatellite geoid suffer from the same cumulative deterioration away from the starting point as the classical astrogeodetic geoid.

Also, one meter accuracy in a geoid from the integration of Equation (2) requires that standard errors in the determined astronomic latitude and longitude should be less than 1 arc second and systematic errors be less than  $0''2$  [Bomford, 1962]. Presently, such accuracies cannot be achieved at sea. The absolute accuracy of Startracker for astrogeodetic applications has not yet been determined. The dependency of the Startracker on the ship's inertial navigation system (SINS) and methods of updating the SINS cause the Startracker outputs not to be truly astronomic. In the background of all this is the problem of kinematic geodesy [Moritz, 1967, 1971b] -- the separation of gravitational and inertial forces.

## 2.2 Inertial Geoid

Various authors such as Bradley, et al [1966], Schultz, et al [1967], Bradley [1970], Butera, et al [1970] have discussed the use of inertial navigators for determination of the deflection of the vertical at sea. Externally provided geodetic fixes from some other systems such as LORAC or Navigation Satellite are required. The deflections and resultant geoid from this technique are basically similar to the classical

astrogeodetic type. The only difference is that the direction of the gravity vector is determined by SINS instead of by astronomical observations.

First, it should be pointed out that the geodetic datum of these external reference control systems such as LORAC is not in absolute position and unless the necessary accurate transformation parameters are available and the transformations executed the deflections and hence the geoid so determined are relative. Second, the absolute accuracy of these external reference controls, relative to any selected datum, remains unknown. Other disadvantages of this technique for deducing (not measuring) the deflections of the vertical include dependency on inaccurately known systems and measurement dynamics, statistical modeling of error sources, poor choices of a priori statistics, initial condition information, ill-defined determination of when performance is optimal and utilization of an adaptive filter when optimality does not exist, all of which are involved in Kalman filtering and optimal smoothing used in the deductions. Therefore, an "inertial geoid", in addition to its poor accuracy, is not compatible with the geoid deducible from satellite altimetry.

### 2.3 The Gravimetric Geoid

For a detailed and expert treatment of the gravimetric geoid and its ramifications, the reader is referred to Chapters 2 and 3 of Heiskanen and Moritz [1967], in particular, and to Uotila [1960] for practical computations.

As before, the geoid or undisturbed mean sea level is depicted as a surface by determining its departure,  $N$ , from a regular reference ellipsoid. However, in this case, by implication of the mathematical structure and the field measurements involved, the reference ellipsoid and the geoid are in absolute position. In Figure 1,  $g_p$  is the gravity vector at point  $P$  on the geoid and  $\gamma_A$  is the normal gravity vector at  $A$  on the ellipsoid. A vector is characterized by magnitude and direction. The difference in direction between the two vectors is the deflection of

the vertical. In the astrogeodetic methods, the direction of  $g_p$  was furnished by the station's astronomical latitude and longitude. For all practical purposes this direction is a constant and a function of position. The direction of  $\gamma_A$  or the ellipsoidal normal defined by the geodetic latitude and longitude of A is arbitrary and completely dependent on the shape, position, and orientation of the reference ellipsoid. The difference in magnitude,  $\Delta g$

$$\Delta g = g_p - \gamma_A \quad (7)$$

is termed the gravity anomaly. It is related to the geoidal undulation,  $N$  (Figure 1), according to the famous Stokes' formula or integral and in principle implies integrating Equation (8).

$$N = \frac{R}{4\pi G} \iint_{\sigma} \Delta g S(\psi) d\sigma, \quad (8)$$

where

$R$  = the mean earth radius

$G$  = the mean value of gravity over the earth

$S(\psi)$  = Stokes' function

$\psi$  = the spherical distance between the fixed point (say P) and the variable surface element  $d\sigma$

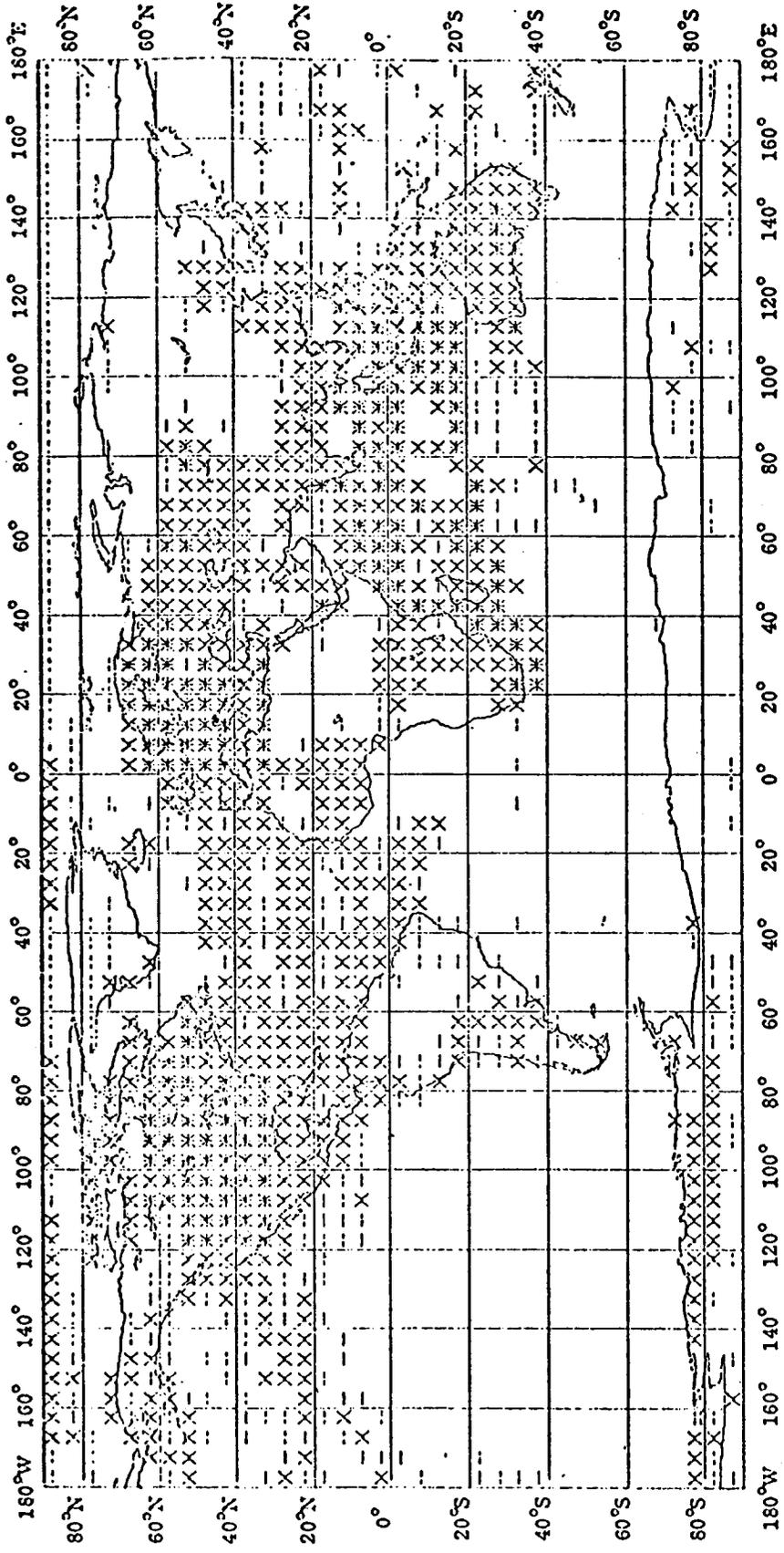
$\sigma$  = surface of the sphere of radius  $R$  with center at the center of gravity.

$$S(\psi) = \frac{1}{\sin(\psi/2)} - 6 \sin \frac{\psi}{2} + 1 - 5 \cos \psi - 3 \cos \psi \ln \left( \sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} \right).$$

The utilization of Equation (8) implies among many other things that:

- (1)  $\Delta g$  is known everywhere on the earth
- (2)  $g_p$  is measured on the geoid or its equivalent is deducible.

Owing to economics and world politics,  $\Delta g$  is not known all over the earth. Predicted values by interpolation or extrapolation are used for areas in which measured values are not available. Figure 2, taken



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FIGURE 2. DISTRIBUTION AND QUALITY OF UNCLASSIFIED TERRESTRIAL GRAVITY DATA  
 5° x 5° MEAN ANOMALY COVERAGE (Rapp, 1970b)

from Rapp [1970b] shows the distribution of and quality of unclassified terrestrial gravity data. In addition,  $g_p$  is hardly ever measured at the geoid. Actual measurements are made on the surface of the earth and reduced to their geoidal equivalents by empirical methods. Some of the parameters involved in the reduction, e.g., crustal density, are represented by intelligent guesses. To avoid the hypothetical assumption about the density, Molodenskii, et al [1962], formulated the concept of the quasi-geoid, and Hirvonen [1960], the telluroid. These substitute surfaces for the geoid will not be further considered for reasons given earlier.

There are many types of gravity reduction methods. Each method results in a slightly different type of gravity anomaly. Furthermore, with reference to Equation (8), the function or anomalous potential,  $T$ , given by Heiskanen and Moritz [1967], as

$$T = \frac{R}{4\pi} \iint_{\sigma} \Delta g S(\psi) d\sigma \quad (9)$$

is assumed to be harmonic outside the geoid. Therefore, the effect of terrestrial masses outside the geoid, or undisturbed mean sea level, must be removed by a suitable gravity reduction method. After the reductions are made, the derived geoid is slightly changed and is termed a "regularized geoid" or "co-geoid". Accordingly, there are as many co-geoids as reduction methods and theories used. The "free-air co-geoid" most nearly coincides with the actual geoid.

In its original form (Equation 8), the Stokes' integral requires also that the reference ellipsoid should (1) have the same potential as the geoid and (2) enclose the same mass as the actual earth. These two requirements are never fulfilled.

The gravimetric geoid as determined by the original Stokes' integral (Equation 8) is not only in absolute position but also has "true" shape, unlike the various categories of astrogeodetic geoids. However, it lacks proper scale. This scale error has been assessed by various experts as ranging from 10 m to 50 m.

### 2.31 Scaling the Gravimetric Geoid From Stokes' Integral

A detailed exposition of this is given in Heiskanen and Moritz, [1967]. The theoretical step to providing this scale is to generalize Stokes' formula for geoidal undulation,  $N_a$ , to hold for any arbitrary reference ellipsoid whose center coincides with the center of the earth. The generalized formula is of the form

$$N_a = \frac{K\delta M}{RG} - \frac{\delta W}{G} + \frac{R}{4\pi G} \iint_{\sigma} \Delta g S(\psi) d\sigma \quad (10)$$

or

$$N_a = N_o + \frac{R}{4\pi G} \iint_{\sigma} \Delta g S(\psi) d\sigma \quad , \quad (11)$$

where

$$N_o = \frac{K\delta M}{RG} - \frac{\delta W}{G} \quad (11a)$$

$\delta M$  = exact mass of the earth minus the mass of the ellipsoid in use

$\delta W$  = potential of the geoid minus that of the ellipsoid

$K$  = Newtonian gravitational constant.

The right side of Equation (11) differs from that of (8) by the term  $N_o$  termed the zero-order undulation [Rapp, 1967]. If both  $\delta M$  and  $\delta W$  were known accurately, application of Equation (11) would give the geoid in absolute position and with proper scale. In Heiskanen and Moritz [1967], Rapp [1967], and Fubara [1969], various approaches to the determination of  $N_o$  are given, but it is still a formidable problem and the gravimetric geoid is now not generally known accurately to within 10 to 20 meters in the oceans.

Very surprisingly, in most published gravimetric geoids, the issue of proper scale is completely ignored. This scaling can be shown to be equivalent to changing the equatorial radius of the reference

ellipsoid on which the gravity anomalies used in Equation (8) are based [Heiskanen and Moritz, 1967]. From gravity data alone, the scale of the geoid can never be determined. Because of incomplete global coverage of observed gravity, predicted  $5^\circ \times 5^\circ$  gravity anomalies whose standard errors are estimated at  $\pm 20$  mgals to  $\pm 50$  mgals and higher are often used. In the face of these, it is surprising that anyone can compute through the use of Stokes' integral an absolute geoid of  $\pm 2$  m accuracy.

An alternative to the use of Stokes' integral is to compute, from gravity anomalies all over the earth, the meridian and prime vertical components of the deflection of the vertical  $\xi$  and  $\eta$ , respectively, through the use of Vening Meinesz formulas. The abbreviated form of these formulas is

$$\xi = \frac{1}{4\pi G} \iint_{\sigma} \Delta g \frac{ds}{d\psi} \cos \alpha d\sigma \quad (12)$$

$$\eta = \frac{1}{4\pi G} \iint_{\sigma} \Delta g \frac{ds}{d\psi} \sin \alpha d\sigma \quad (13)$$

the theoretical and computational details of which can be found in Heiskanen and Moritz [1967], and Uotila [1960]. The  $\xi$  and  $\eta$  so obtained are absolute, i.e., referenced to the earth's center of mass. Thereafter,  $\epsilon$  can be computed according to Equation (3) and the geoidal undulation computed from Equation (2).

Unlike Stokes' integral, Vening Meinesz formulas are valid for any arbitrary reference ellipsoid. However, they also require the use of gravity anomalies all over the earth, and in particular a dense gravity net around the computation points.

All the deficiencies in theory, data quality and quantity in gravimetric geodesy are extensively discussed in Chapter 7 of Heiskanen and Moritz [1967]. These deficiencies have led to many unanswered questions about the accuracies of gravimetrically computed geoidal undulations and deflections of the vertical. A few of the numerous efforts

addressed to these unresolved issues include Kaula [1957, 1959, 1966], Groten and Moritz [1964], Heiskanen and Moritz [1964], Moritz [1962, 1966, 1969, 1971, 1971a], Shaw, et al [1969], Henrikson, et al [1970], Rapp [1970a, 1970b]. The most important source of disagreement is on statistical modeling and estimation recognized by all to be indispensable in efficient gravity data analysis.

Consequently there is general disagreement on all or some of these:

- (1) Estimation of interpolation and extrapolation errors of the gravity anomaly,  $\Delta g$
- (2) Estimation of the effects of these errors on the derived  $N$ ,  $\xi$ , and  $\eta$
- (3) Determination of the best prediction method
- (4) Estimation of the effect of neglected distant zones

in the works of Molodenskii, Kaula, Moritz, Henrikson, and Rapp. Figure 3, taken from Groten and Moritz [1964] depicts the standard errors  $M_N$  due to neglect of distant zones beyond a radius of  $\psi_0$  from the computation points of gravimetrically computed geoidal undulation using the improperly scaled Stokes' integral. The computation is for latitude  $45^\circ$  and global gravity anomaly distribution of one point per blocks of  $n^\circ \times n^\circ$ ,  $n$  being the numbers shown on the graphs. A comparable computation in Molodenskii, et al [1962], gives values about 70 percent larger.

Perhaps the biggest source of systematic scale error in gravimetric geoidal profiles through the use of Vening Meinesz formulas is that an initial point ( $N_A$  in Equation (2)) at which the correct absolute value of the geoidal undulation is known must be specified. Such a value is hardly known accurately anywhere. Any geoid based on gravity data alone is therefore not suitable for the geodetic absolute calibration or verification of the geoid deducible from satellite altimetry.

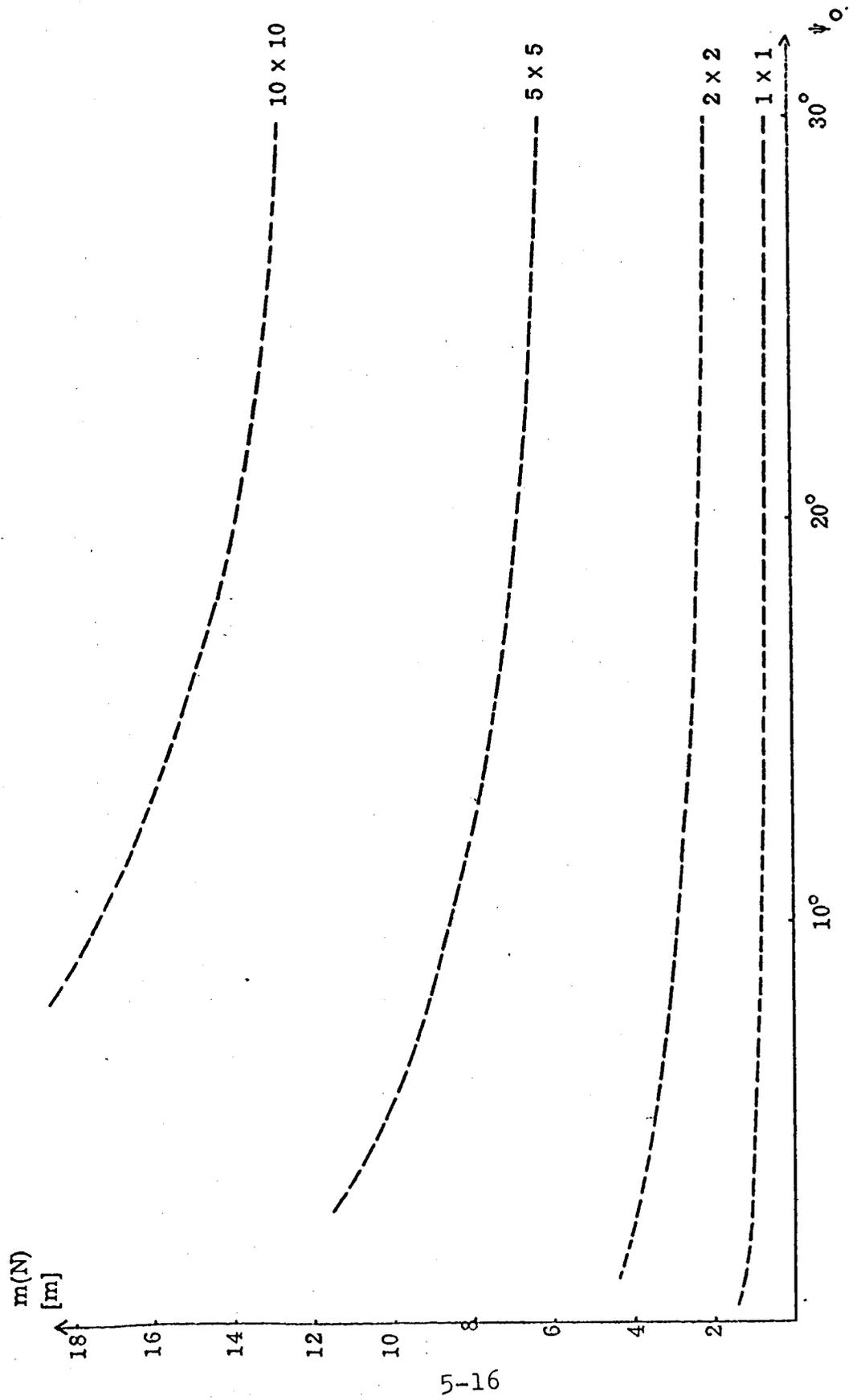


FIGURE 3.  $m(N)$  FROM POINT MEASUREMENTS AS A FUNCTION OF  $\psi_0$ .

## 2.4 Satellite Geoids

The dynamics of artificial satellite motions around the earth can be used for (a) a geopotential or (b) a dynamic geometric computation of the geoid.

### 2.41 Geopotential Satellite Geoid

Satellite orbits are influenced by the irregularities of the earth's gravity field, which are usually expressed in terms of a development in spherical harmonics [Moritz, 1964]. The spherical harmonic coefficients can be determined from the analysis of known satellite orbits or from gravity measurements all over the earth's surface. The undulations of the geoid can be computed from those spherical harmonic potential coefficients [Bursa, 1968, 1969], [Bacon, 1970], [Moritz, 1964], [Rapp, 1970a].

At satellite heights, this technique cannot detect small-scale features of the geoid but only the general outline. All the satellite geoid so far computed by this technique differ in details by about 10 to 80 meters. The technique has a fundamental drawback. On the one hand, the gravity field, i.e., the potential coefficients, must be known for precise prediction of satellite orbits. On the other hand, and ironically, the computation of the coefficients depend on analysis of pre-known satellite orbits.

The summary of the various modes of this technique in Rapp [1970a] also contains implicit drawbacks of the technique. The recommendation to use Method 1 of that reference by setting the zero order undulation  $N_0$  to zero unfortunately gives, as in the original Stokes' integral, a scaleless geoid because the undulations so obtained will refer to some ellipsoid of unknown size but which has the same mass as the earth and whose surface has the same potential as the geoid, whatever the unknown mass of the earth and the unknown potential of the geoid may be. In view of this, the equality of the results in Table 3 of

Rapp [1970a] raises an important question. This equality is questionable because it implies that his Equation (6) or our Equation (11a) must be truly zero which means that his assigned constants for the geocentric gravitational constant, angular rotational velocity of the earth, the flattening and equatorial radius of the reference ellipsoid, the potential of the geoid must be the true values.

Besides the theoretical problem about the convergence of the series in spherical harmonic expansion [Heiskanen and Moritz, 1967], [Moritz, 1971a], the poor quality of these coefficients are often overlooked in spite of values such as  $C_{6,2} = 0.0283$  with standard error of  $\pm 0.0396$  [Rapp, 1969]. It is often argued that the quality of each individual coefficient does not critically affect the quality of the set of coefficients as a whole. How can coefficients be unreliable individually and yet be accurate collectively unless they have equal cancelling errors?

#### 2.42 Geometric Satellite Geoid

The geoidal undulation,  $N$ , the orthometric height,  $H$ , and the ellipsoidal height,  $h$ , are related according to Equation (6). The absolute space rectangular coordinates,  $x$ ,  $y$ ,  $z$  of a station can be deduced from "dynamic satellite resection". From iterative procedures as in Heiskanen and Moritz [1967],

$$h = (x^2 + y^2)^{1/2} \sec\varphi - \nu \quad , \quad (14)$$

where

$\varphi$  = geodetic latitude of resected point

$\nu$  = prime vertical radius of curvature of the reference ellipsoid in use for the resected point

$h = H + N$  (as in Equation (6)).

On land,  $H$  is deduced from spirit leveling and gravity measurements. On the geoid or mean sea level  $H$  is zero. Thus the deduction of  $h$  at sea gives the geoidal undulation,  $N$ , to within the accuracy of the separation of the sea surface topography and the actual geoid.

The use of this technique at sea is under investigation [Mourad and Fubara, 1971a], [Martin, et al, 1971], [Stanley, et al, 1971]. From ships positioned over ocean bottom acoustic transponders, this technique can be effectively implemented. If geostationary and orbiting satellites of accurately known geocentric coordinates are available, ranging systems such as laser or C-band radar can be used in a geometric solution.

## 2.5 Combination Geoids

Two types of combination geoids exist. One is from a combination of satellite and terrestrial data such as gravity, triangulation, and astronomic observations. Some works along this line are Kaula [1961, 1966], Mather [1970, 1971]; Rapp [1970c], Heiskanen and Moritz [1967], Yeremeev, et al [1971], and Fubara [1971]. The fundamental problem is establishing practical and efficient mathematical and statistical models that give stable solutions in generalized least squares adjustments of these hybrid data. There is no doubt that this combination has to be effected in order to resolve the problems of scale, shape, and orientation on a global basis for the geoid and interrelation of various geodetic datums. The method is usable both on land and also at sea in the light of results in Mourad, et al [1970a, 1970b, 1971a,b] and Fubara, et al [1971]. However, it is more complex, less economical, more time consuming, more suited to broad features of global geoid mapping, and much less accurate or suitable for detailed local mapping of the geoid as required for altimetry sea-truth than astrogravimetry.

The second method is termed astrogravimetry [Molodenskii, et al, 1962], [Heiskanen and Moritz, 1967]. It is basically a combination of all the desirable features of the astrogeodetic and gravimetric computations of the geoid. At the same time, it is not affected by any

of the disadvantages of either method and particularly it does not require complete global coverage of gravity data as the influence of distant zones is not important. The technique is applicable at sea but the accuracy achievable at sea will depend on the reliability of the systems for astronomical and geodetic coordinates measurements.

The astrogravimetric geoid acquires correct shape and absolute orientation from the gravity data employed. It obtains correct scale from the astrogeodetic parameters. It is highly suitable and accurate for mapping local details of the geoid. It is speedy and economical because it requires only a dense local gravity-net in the test area alone.

### 3.0 SATELLITE ALTIMETRY "GEOID"

Figure 4 is a representation of a cross section containing a satellite altimeter orbit and some surfaces associated with satellite altimetry. Satellite altimetry is faced with several problems including the effective "hardware" calibration of the range TM and the physical definition of the surface, M, which is some mean surface defined by the altimeter ranges but whose exact position relative to either the geoid, G, or some mean sea surface, S, at any instant of time is currently unknown.

The interrelationships between the surfaces E, G, and S can be handled in test areas. There are analytical procedures in combination with "sea-truth" data by which a geoid can eventually be computed from satellite altimetry data. The solutions for these problems are not the subject of this paper. Subject to the accuracies of computed satellite positions and the altimeter calibration, the geoid so deduced should be in absolute position (i.e., centered at the earth's center of mass) and should have proper scale, shape, and orientation. The benefits of the success of this mission have been widely publicized in such as Greenwood, et al [1969], Koch [1970], Lundquist [1967], NASA [1970], Young [1970], Stanley, et al [1971], and Kaula [1970].

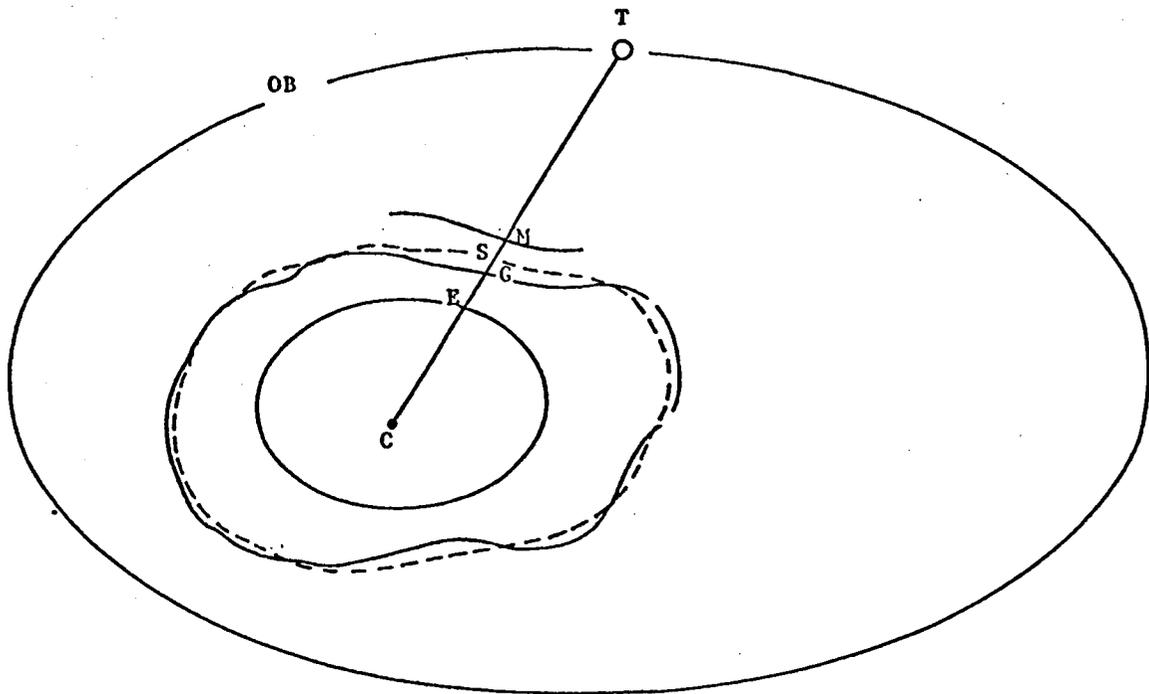


FIGURE 4. REPRESENTATION OF SURFACES ASSOCIATED WITH SATELLITE ALTIMETRY

- C = Earth's Center of Gravity
- E = Surface of a Geocentric Reference Ellipsoid
- G = Geoid (the undisturbed Mean Sea Level)
- S = Mean Instantaneous Sea Surface (MISS)
- OB = Mean Satellite Orbit
- T = Satellite Altimeter at an Instant
- M = An Arbitrary Surface Defined by a "Hardware" Calibrated Altimeter

Based on all the foregone discussions, it is proposed to outline the conditions and the practical way for computing, in test sites,

geoidal profiles that are compatible in scale, shape, and orientation with the geoid deducible from satellite altimetry, so that geodetic processing of satellite altimetry data for computing the true geoid can be accurately effected.

#### 4.0 COMPATIBILITY REQUIREMENTS

The determination in several test areas of "sea-truth" [Weiffenbach, 1970], [Raytheon, 1970], or segments of the absolute marine geoid will serve two main purposes from the geodetic point of view: (1) calibration and evaluation of the satellite altimeter; (2) as controls required for the geodetic analytical processing of satellite altimetry to determine the absolute marine geoid. To achieve these two goals, the sea-truth must have true scale, true shape, and true absolute orientation.

In geodesy, these conditions mean that (a) the center of the reference ellipsoid (equatorial radius,  $a$ , and flattening,  $f$ ) employed in geoidal computations must coincide with the earth's center of mass and (b) the minor axis of the reference ellipsoid must coincide with the mean rotation axis of the earth so that the geoidal undulation,  $N$ , the meridian and prime vertical components of the deflection of the vertical,  $\xi$  and  $\eta$ , are absolute. These five parameters,  $a_0$ ,  $f_0$ ,  $N_0$ ,  $\xi_0$ ,  $\eta_0$ , as used in all local geodetic datums do not satisfy these conditions. Therefore, geoids based on different local datums are incompatible until they have been reduced to the same geocentric system based on a single "general terrestrial ellipsoid". Unfortunately, the parameters required in such reductions are currently too inaccurately known for use in geodetic calibrations or controls. The necessary correction parameters such as (1) datum shifts, (2) datum tilts of some major geodetic (local) datums have been computed, for example, by Veis [1965, 1968], Lambeck [1971]. The uncertainties in these geodetic parameters can be up to 70 meters or more as shown in Tables 1 and 2.

TABLE 1. DATUM SHIFTS FOR MAJOR GEODETIC DATUMS IN CARTESIAN COORDINATES  
(IN METERS)

Datum	Veis [1965]*			$\sigma$	Veis [1968]**			Lambeck [1971]***		
	$\Delta X$	$\Delta Y$	$\Delta Z$		$\Delta X$	$\Delta Y$	$\Delta Z$	$\Delta X$	$\Delta Y$	$\Delta Z$
NAD	- 2	+ 119	+ 229	10	- 26	+ 155	+ 155	- 31.8 ± 8.0	178.0 ± 12.2	177.6 ± 11.9
HAW	+ 12	- 145	- 19	24	+ 59	- 263	- 203	--	--	--
EUR	- 70	- 93	- 115	14	- 93	- 132	- 143	- 64.5 ± 19.0	-154.8 ± 11.0	- 46.2 ± 17.5
CEN	+ 45	- 74	+ 41	9	--	--	--	--	--	--
JAP	- 9	+ 626	+ 727	18	- 140	+ 510	+ 683	--	--	--
ARG	- 117	- 155	+ 30	27	- 167	+ 128	+ 25	-320.2 ± 12.1	203.8 ± 24.1m	391.7 ± 7.5
IND	+ 227	- 769	+ 216	33	+ 293	+ 697	+ 228	--	--	--

$\frac{1}{f}$

- \* Veis [1965] 6,378,169 m. ± 8 m. 298.25
- \*\* Veis [1968] 6,378,142 m. ± 6 m. 298.255 ± 0.005
- \*\*\* Lambeck [1971] 6,378,155 m. 298.25

UT  
1  
2  
3

TABLE 2. CORRECTIONS TO THE ADOPTED DEFLECTION OF THE VERTICAL AND GEODAL UNDULATION AT DATUM ORIGIN

Datum	Veis [1965]		$\sigma$	Veis [1968]		Lambeck [1971]	
	$\delta\eta$	$\delta N$		$\delta\eta$	$\delta N$	$\delta\eta$	$\delta N$
NAD	- 0.60	+ 0.62	± 4.2m	- 0.23	+ 1.61	- 28 meters	- 1.60 ± 0.11 - 42.2m ± 3.3
HAW	+ 6.28	- 4.51	± 24	+11.85	- 8.55	0	--
EUR	+ 2.84	+ 2.41	± 14	+ 2.61	+ 3.45	+ 3	" - 5.79 0.5m
CEN	- 1.30	+ 2.75	± 9m	--	--	--	--
JAP	-13.44	+15.30	± 8m	-11.85	+ 9.63	+ 31	--
ARG	- 0.05	+ 1.00	± 27m	+ 0.15	+ 2.85	+ 44	- 13.3 - 6.4
IND	- 0.11	+ 3.44	± 33m	- 1.35	+ 4.40	- 64	- 70m

where

NAD = North American Datum (1927)

HAW = Hawaiian Datum

EUR = European Datum

CEN = Australian Datum (1963)

JAP = Japanese Datum

ARG = Argentinian Datum

IND = Indian Datum

and

X-axis = Longitude  $0^\circ$

Y-axis = Longitude  $90^\circ$  E

Z-axis = Earth mean rotation axis (mean pole of 1900-1905)

The corrections

$\delta\xi_0$  for meridian component of deflection of vertical

$\delta\eta_0$  for prime vertical component of deflection of the vertical

$\delta N_0$  for height of geoid above the ellipsoid

are due to purely translatory corrections to the geocenter to satisfy condition (a) above. The corresponding datum tilts to fulfill the parallelity requirements  $\Delta\xi$ ,  $\Delta\eta$ ,  $\Delta N$  are shown in Table 3.

TABLE 3. DATUM TILTS

Datum	Veis [1968]			Lambeck [1971]	
	$\Delta\xi$	$\Delta\eta$	$\Delta N$	$\Delta\xi$	$\Delta\eta$
NAD	1''5	0''2	-2	-0''62 $\pm$ 0''50	-0''53 $\pm$ 0''5
EUR	1''6	-1''2	+10	2''2 $\pm$ 0''7	1''4 $\pm$ 0.6

It needs to be emphasized that the shifts are for the specified local datum origins. Quantitatively, any other arbitrary point is affected slightly differently. A thorough exposition on this subject can be found in Heiskanen and Moritz [1967]. However, the derivations in that reference are for the ideal case when the absolute orientation parameters

$$\Delta\xi = \Delta\eta = \Delta N = 0 \quad (15)$$

i.e., that the tilts at the datum origin are zero, implying that the minor axis and the major axis of the reference ellipsoid of each datum are strictly parallel to the mean rotation axis and the mean equator of the earth respectively. Under these conditions, a geoid can be transformed from one datum to another by a change of  $\delta a$ ,  $\delta f$ ,  $\delta\xi_0$ ,  $\delta\eta_0$ , and  $\delta N_0$  in the initial parameters of the datum origin of the geoid. The corresponding corrections of  $\delta\xi$ ,  $\delta\eta$ , and  $\delta N$  to the values of the deflection components and the geoidal undulation at any arbitrary point are given by

$$\begin{aligned} \delta\xi = & (\cos\varphi_0 \cos\varphi + \sin\varphi_0 \sin\varphi \cos\Delta\lambda) \delta\xi_0 - \sin\varphi_0 \sin\Delta\lambda \delta\eta_0 \\ & - (\sin\varphi_0 \cos\varphi - \cos\varphi_0 \sin\varphi \cos\Delta\lambda) \left( \frac{\delta N_0}{a_0} + \frac{\delta a}{a_0} + \sin^2\varphi_0 \delta f \right) \\ & - 2\cos\varphi (\sin\varphi - \sin\varphi_0) \delta f \quad . \end{aligned} \quad (16)$$

$$\begin{aligned} \delta\eta = & \sin\varphi_0 \sin\Delta\lambda \delta\xi_0 + \cos\Delta\lambda \delta\eta_0 + \cos\varphi_0 \sin\Delta\lambda \left( \frac{\delta N_0}{a_0} + \frac{\delta a}{a_0} + \right. \\ & \left. + \sin^2\varphi_0 \delta f \right) . \end{aligned} \quad (17)$$

$$\begin{aligned} \delta N = & a_0 \{ (\sin\varphi_0 \cos\varphi \cos\Delta\lambda - \cos\varphi_0 \sin\varphi) \delta\xi_0 + \cos\varphi_0 \sin\Delta\lambda \delta\eta_0 \} \\ & + (\sin\varphi_0 \sin\varphi + \cos\varphi_0 \cos\varphi \cos\Delta\lambda) (\delta N_0 + \delta a + a_0 \sin^2\varphi_0 \delta f) \\ & - \delta a + a_0 (\sin^2\varphi - 2 \sin\varphi_0 \sin\varphi) \delta f \end{aligned} \quad (18)$$

Where

$\varphi_0$  and  $\lambda_0$  = geodetic latitude and longitude of the datum origin, respectively

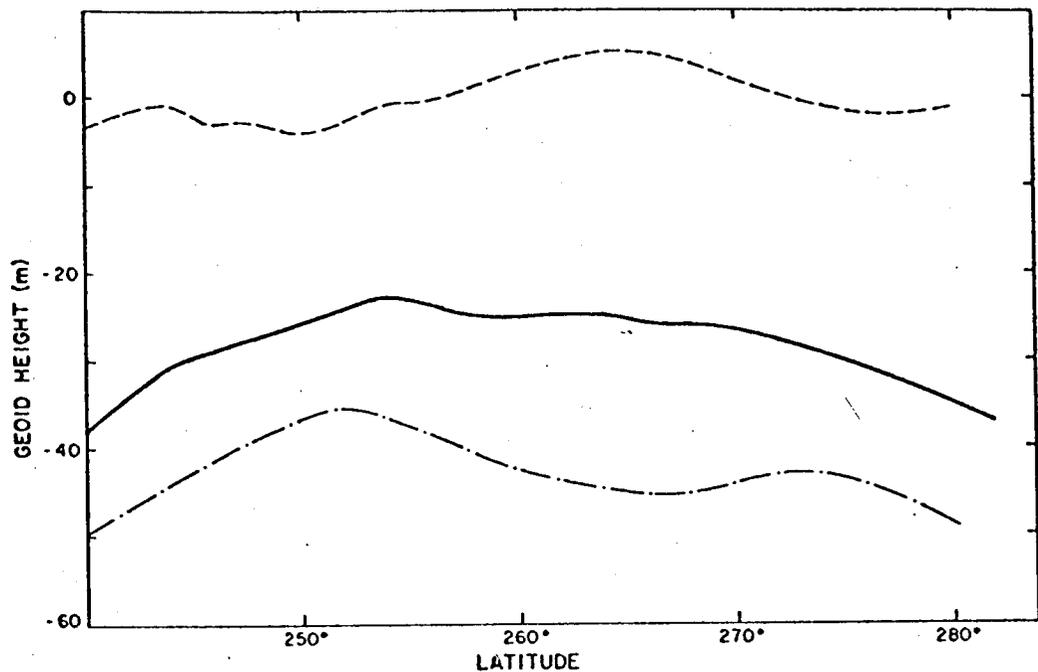
$\varphi$  and  $\lambda$  = geodetic latitude and longitude, respectively of any arbitrary point in that geodetic system

$$\Delta\lambda = \lambda_0 - \lambda$$

$$\begin{array}{l} \delta a = a_0 - a \\ \delta f = f_0 - f \end{array} \left\{ \begin{array}{l} \text{Parameters of the old reference} \\ \text{ellipsoid minus those of the new one.} \end{array} \right.$$

The absolute orientation vectors can be determined by either analytical reconciliation of gravimetric deflections (or undulations) and astrogeodetic deflections (or undulations) of corresponding stations [Mather, 1970] or by satellite geodesy techniques in combination with terrestrial data. They should be corrected for implicitly as in Lambeck [1971] or as explicit rotation corrections before computing at the datum origin the shift dependent  $\delta\zeta_0$ ,  $\delta\eta_0$ , and  $\delta N_0$  involved in Equations 16, 17, and 18. However, the accuracies with which these tilts can be determined on a global scale is still questionable due to measurement errors, inaccuracies in orbital dynamics computations, and quantity and global distribution of available data.

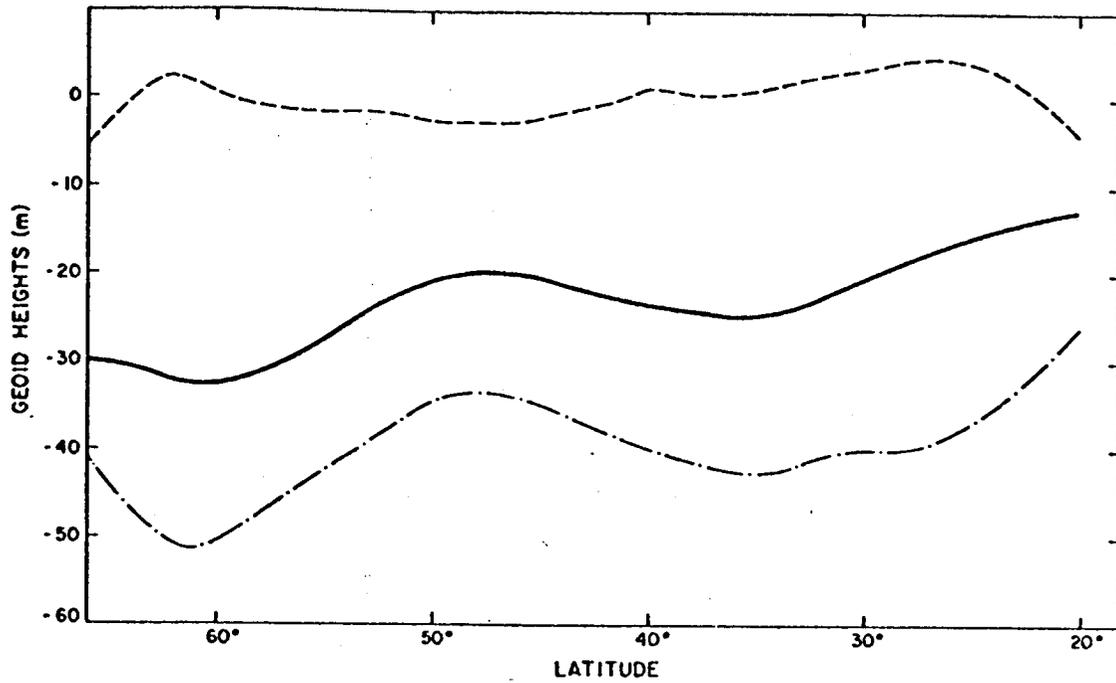
The various problems and inaccuracies involved in trying to reconcile various geoids on different datums on a global basis are discussed by Fischer, et al [1968]. The title of that paper, "New Pieces in the Picture Puzzle of an Astrogeodetic Geoid Map of the World" truly tells it as it is. One of the conclusions of that paper, "If one expects a geodetic accuracy of a few meters, the variety of numbers is bewildering", is still valid today. Figures 5a, b, and c taken from from Gaposchkin and Lambeck [1969] amplify the magnitudes of the incompatibility between astrogeodetic geoids (Section 2.1) and combination geoids (Section 2.5) even after all necessary translations and rotations



$\phi = 35^\circ$

a

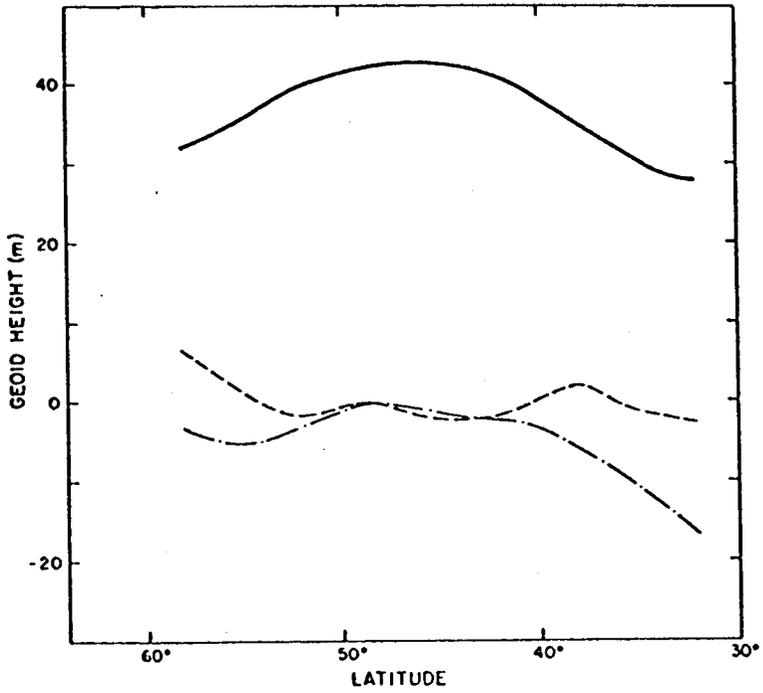
FIGURE 5 a, b, c. COMPARISONS BETWEEN GEOID PROFILES OBTAINED FROM THE COMBINATION SOLUTION (SOLID LINES) WITH PROFILES OBTAINED FROM ASTROGEODETTIC MEASUREMENTS TRANSFORMED INTO THE GLOBAL REFERENCE SYSTEM (DASHED LINES). THE DIFFERENCE BETWEEN THE TWO PROFILES, AFTER THE SYSTEMATIC PART HAS BEEN SUBTRACTED, IS INDICATED BY THE DOTTED LINE.



$\lambda = 260^\circ$

North American Datum

5b



$\lambda = 16^\circ$   
European Datum

5c

have been performed to make them compatible. These figures refer to North American and European datums which supposedly have the best accurate geodetic data and computations.

The requirements of scale, shape, and orientation and expediency in practical determinations in test sites for geodetic sea-truth required by satellite altimetry rule out the applicability of any of the categories of geoid described except the astrogravimetry.

#### 5. MARINE GEOID BY ASTROGRAVIMETRY

It has been shown how and why a marine geoid by astrogravimetry meets accurately the compatibility requirements in scale, shape, and absolute orientation required for satellite altimetry processing. A brief outline of the marine operations needed is as follows.

At any chosen test site marine geodetic controls using geodetic-acoustic techniques are established [Mourad, et al, 1970 a, b, c, d], [Fubara, et al, 1971] at say about 100 to 150 km intervals. Over these control points several repeated measurements of astronomic latitudes and longitudes are made to about 1 to 2 arc seconds accuracy. The corresponding geodetic latitude, longitude, and height are accurately and repeatedly measured over the same control points. At each geodetic control point, both the astronomic and geodetic measurements are reduced to a single point using techniques as in Mourad, et al [1970b], which continuously determine accurately the ship's position, speed, and heading relative to the geodetic ocean bottom markers.

In the test site and its surroundings, a dense gravity net of profile runs at about 10 to 20 km intervals should be conducted. The geodetic control points already established at the site should be linked up with gravity profile runs. At the same time, these control points will serve as base stations for the gravity profile runs and also furnish highly accurate ground ship speed and heading needed in the

gravity data reduction [Kaula, 1970]. The astronomic, geodetic coordinates, and gravity data are then processed together to give accurate details of the marine geoid at the test site.

## 6. CONCLUSION

The need for ocean surface mapping and the eventual determination of the absolute geoid at sea and the peripheral benefits to geodesy, oceanography, space research, marine environmental control, prediction, and resources exploitation is widely recognized. Satellite altimetry is expected to meet this need. The success of satellite altimetry depends on factors including adequate sea-truth. It has been shown that geodetic determination of certain features of the sea-truth is indispensable, and that gravity measurements alone cannot meet the requirements. Astrogravimetry is suggested as the most speedy, economical, and reliable answer. The implementation of astrogravimetry at sea is well within the current state of the art.

Furthermore, it is necessary to determine from satellite altimetry an absolute geoid and not a relative geoid because there are more than enough relative geoids already computed. These relative geoids cannot satisfy many of the needs of geodesy, oceanography, and earth-gravity modeling. Without the use of absolute geoid profiles as controls in the geodetic processing of satellite altimetry data, a relative geoid will be the result. In view of the foregone discussions, should more funds and efforts be spent to determine yet another relative geoid without proper scale, shape, and absolute orientation? Table 4 contains a summary of the findings of the paper.

TABLE 4. COMPARISON OF CONVENTIONAL MARINE GEOIDS AND SATELLITE ALTIMETRY "GEOID" FOR COMPATIBILITY

Type of Geoid	Compatibility Criteria			Quality of Geoid and Sources of Deficiencies
	Absolute Orientation	Correct Scale	True Shape	
(1) Astrogeodetic (classical)	No	Yes	False tilt	Detailed local geoid highly dependent on density and accuracy of deflection stations. Rapid error accumulation. Bad local datum influence. Currently not expedient at sea. Not compatible.
(2) Astrosatellite	Yes/No	Yes	Possible	Currently poor accuracy at sea as geoid details need highly accurate and dense data distribution. Suitable for evaluation but not absolute calibration of Sat. Alt. "Geoid".
(3) Inertial	No	Not reliable	Not reliable	Very poor accuracy, deficiencies in theory for data deduction. Accurate external geodetic reference required in navigation mode. Not compatible with Sat. Alt. "Geoid".
(4) Gravimetric (a) Stokes'	Yes	No	Possible	Not for ABSOLUTE CALIBRATION but good for shape evaluation. Needs adequate global coverage of data; theory problems in data prediction and reduction. Compatible in shape and orientation only but not in scale.
(b) Vening Meinesz	Yes	Possible dependent on initial point	Possible	More dependent on dense local gravity net and less influenced by distant zone data which are still needed. Problems in prediction and reduction theories. Compatible in shape and orientation but correctness of scale dependent on assumed initial point.
(5) Satellite (a) Geopotential coefficients	Yes	Dependent on method used	General outline	Poor coefficient accuracy, inadequate for geoid details. Not suitable for calibration of Sat. Alt. "Geoid".
(b) Geometric/dynamic	Possible	Yes	Possible	Highly dependent on orbit accuracy and geometry. Could provide in the future compatible detailed geoid profiles.
(6) Combined Satellite/Terrestrial, Astronomic, Geodetic, Gravity	Yes?	Yes?	Yes?	Development of techniques in progress. Theoretically, could provide global geoid using <u>world-wide data</u> coverage. Not suitable for local geoid details as required for satellite altimetry test areas.
(7) Astrogravimetry	Yes	Yes	Yes	Requires ONLY LOCAL GRAVITY data, speedy and economical. BEST suitable for geoid details in Test Areas. COMPATIBLE with expected Sat. Alt. "Geoid" in scale, shape, and orientation.
(8) Satellite Altimetry	Yes	Yes	Yes	Development in progress. If successful, provides the best hope currently for speedy, economical determination of global marine geoid with sufficient accuracy and details to meet oceanographic, geodetic, space programs, environmental control and prediction needs.

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